Full-wave-equation depth migration using multiple reflections

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SUMMARY

We present a migration method in the temporal frequency domain that can propagate both downgoing and upgoing waves. The method can handle arbitrary velocity model, and we demonstrate its ability to image overturned events and steep reflectors. Our algorithm has the advantage of migrating the data sequentially in depth and frequency, leading to significant advantages compared to Reverse Time Migration when combined with migration velocity analysis. We also show that the method can easily generate offset and angle gathers.

INTRODUCTION

We present an enhancement to the depth extrapolation algorithm introduced in Sandberg and Beylkin (2009). The method in Sandberg and Beylkin (2009) uses spectral projector as a way to suppress the evanescent waves without modifying the propagating waves. By using spectral projectors, the depth extrapolation scheme propagates both up and downgoing waves, which is necessary to properly image overturned events. Furthermore, no approximations in the mathematical formulation are necessary for laterally varying media.

We note that traditional downward extrapolation algorithms rely on operator splitting to overcome the instability introduced by evanescent waves. Such methods carry two penalties: first, in addition to the evanescent waves, they also suppress the upgoing propagating waves and, second, the mathematical formulation of operator splitting is only exact in a media without lateral variations, thus introducing additional errors if lateral variations are present.

In the algorithm described in Sandberg and Beylkin (2009), we use only the incident wave field as a source, thus ignoring scattered waves acting as secondary sources. In this paper, we modify the algorithm in Sandberg and Beylkin (2009) in order to take a proper account of such secondary reflections recorded at the surface. These modifications further improve imaging of steep and overturned events. Assuming that we can separate the incident and the scattered fields at the surface, the resulting algorithm is a downward continuation method that handles arbitrary medium complexity, propagates both upgoing and downgoing waves, and images steep reflectors and overturned events. We demonstrate the algorithm by migrating data generated for a challenging model and also compute angle gathers that are easily generated using our modified approach.

We note that the imaging condition in Reverse Time Migration (RTM) does use the secondary reflections recorded at the surface. With the modifications described here, our method and RTM now use the same information available at the surface and, as a result, the two methods may now be carefully compared as we plan to do in the near future.

Finally, we note that downward extrapolation algorithms that rely on operator splitting do not treat secondary reflections correctly (due to the suppression of upgoing propagating waves), thus making it difficult, if not impossible, to use such data for imaging.

FULL-WAVE-EQUATION DEPTH EXTRAPOLATION FOR MIGRATION

In this section we briefly summarize the algorithm in Sandberg and Beylkin (2009). We downward continue a wavefield $u(x,y,z,t)$ from $z=z_n$ to $z=z_n+\Delta z$ by solving

$$\frac{\partial^2 \hat{u}}{\partial z^2} = P\left[-\left(\frac{\omega}{v(x,y,z)}\right)^2 - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right] P\hat{u} \equiv PLP\hat{u} \quad (1)$$

with the initial conditions

$$\hat{u}(x,y,z_n,\omega) = f(x,y,\omega) \text{ and } \frac{\partial}{\partial z}\hat{u}(x,y,z_n,\omega) = g(x,y,\omega),$$

where $\hat{u}$ refers to the Fourier transform in time of u and ω denotes the temporal frequency. The operator P in (1) is the spectral projector that maps functions onto the eigenspace of L spanned by the eigenvectors associated with negative eigenvalues.

The introduction of the spectral projector P is the key element of our approach in Sandberg and Beylkin (2009) since without this operator the problem is notoriously unstable due to the presence of positive eigenvalues of L associated with the evanescent waves. Using the spectral projector P allows us to suppress these and only these modes, thus leaving all propagating modes intact. In the medium with a constant background this projector reduces to the Fourier transform followed by a cutoff filter, an algorithm introduced in Kosloff and Baysal (1983).

In order to initialize the downward continuation near the surface $z=0$, we need to estimate the normal derivative at the surface. Assuming constant velocity in the lateral direction at the surface, we compute the normal derivative analytically, as in Kosloff and Baysal (1983) or Sandberg and Beylkin (2009). We then may apply (1) to extrapolate the wavefield from any depth level $z=z_n$ to the next one at $z=z_n+\Delta z$.

To generate an image, we start from the recorded data $u^r(\mathbf{x}_{\mathbf{s_i}},\mathbf{x}_{\mathbf{r}},z=0,t)$ for sources at locations $\mathbf{x}_{\mathbf{s_i}}=(x_{s_i},y_{s_i})$, $i=1,\ldots,n_{source}$ and the initial source wavelet recorded at the same receiver locations, $u^s(\mathbf{x}_{\mathbf{s_i}},\mathbf{x}_{\mathbf{r}},z=0,t)$. As described in Sandberg and Beylkin (2009), we select the initial source wavelet u^s at receiver locations as recorded in a constant background, i.e. the direct arrivals. The generalized imaging condition (for all offsets) for shot-gather migration in this case (mentioned, but not described in detail in Sandberg and Beylkin (2009)), may then be written as

$$I(x,y,z,h_x,h_y) = \sum_{i=1}^{n_{source}} \sum_{k=1}^{n_{freq}} \hat{u}^r(\mathbf{x_{s_i}}, x+h_x, y+h_y, z, \omega_k) \times \overline{\hat{u}^s(\mathbf{x_{s_i}}, x-h_x, y-h_y, z, \omega_k)}, \qquad (2)$$

where $\hat{u}^r$ and $\hat{u}^s$ denote the Fourier transform of extrapolated wavefields with respect to time. The migrated image is obtained by setting the offsets $h_x = h_y = 0$. This imaging condition with a range of offsets may be used to generate offset or angle gathers, e.g. for velocity analysis.

THE MODIFIED ALGORITHM

In Sandberg and Beylkin (2009), we used a source field u^s that only consisted of a recording of the incident part of the wave field originating from a source wavelet and recorded along the surface. However, in order to image overturned events, it is helpful to include secondary reflections and consider them as a part of the source wavefield. This, in fact, is done in RTM, where the source field is generated using the complete background (migration velocity) model.

We propose two approaches to account for secondary reflections within the approach in Sandberg and Beylkin (2009). The first method is similar to that in RTM. We generate the initial source wavelet recorded at the receiver locations, $u^s(\mathbf{x_{s_i}}, \mathbf{x_r}, z = 0, t)$, using the background velocity model and then compute $\hat{u}^s$.

An alternative approach is to avoid modeling by using the recorded wavefield $u^r(\mathbf{x_{s_i}}, \mathbf{x_r}, z = 0, t)$ directly since it already contains both the incident and scattered wavefield. The difficulty now is in separating the incident and scattered wavefield in the measured data. Such separation of the incident and scattered wavefields is relatively simple in the surface seismics as these fields are usually disjoint in time. In any case, there are several methods for such splitting that do not depend on the time separation and based on modeling of the direct arrivals. We do not address these issues here and simply assume that such separation of wavefields is available to us.

In order to use the incident and scattered wavefields in the extrapolation algorithm, we need to evaluate their normal derivatives at the surface $z = 0$. The important distinction in the treatment of these fields is in the selection of the sign of the normal derivative. The normal derivative for the scattered field at the surface $z = 0$ is chosen to be of the opposite sign to that of the incident field, to indicate the direction of propagation.

We summarize the new algorithm as follows:

1. Separate the incident and scattered wave field from the measurement data $u^r(\mathbf{x_{s_i}}, \mathbf{x_r}, z = 0, t)$. We denote these wavefields as u^r_{inc} and u^r_{sca}.
2. Estimate the normal derivatives $\frac{\partial}{\partial z}\hat{u}^r_{inc}$ and $\frac{\partial}{\partial z}\hat{u}^r_{sca}$ at the surface $z = 0$ using the method described in Kosloff and Baysal (1983)
(see also Sandberg and Beylkin (2009)).
3. Downward continue the source wave field by solving (1) with the initial conditions
$\hat{u}^s(x,y,0,\omega) = \hat{u}^r(x,y,0,\omega)$ and
$\frac{\partial}{\partial z}\hat{u}^s(x,y,0,\omega) = \frac{\partial}{\partial z}\hat{u}^r_{sca}(x,y,0,\omega) - \frac{\partial}{\partial z}\hat{u}^r_{inc}(x,y,0,\omega)$.
4. Downward continue the receiver wave field by solving (1) with the initial conditions
$\hat{u}^r(x,y,0,\omega) = \hat{u}^r_{sca}(x,y,0,\omega)$ and
$\frac{\partial}{\partial z}\hat{u}^r(x,y,0,\omega) = \frac{\partial}{\partial z}\hat{u}^r_{sca}(x,y,0,\omega)$.
5. Form the migrated image using (2).

By using the generalized imaging condition (2), we may also compute offset gathers (see Ricket and Sava (2001)). We may also generate angle gathers via the slant stack transform of the offset gathers (see Sava and Fomel (2003)).

The new method handles overturned targets similarily to RTM. However, our approach is more flexible and, in particular, allows us to generate the image for a subset of temporal frequencies and, also, to generate migrated image layer by layer. Hence, we may use our method within a velocity analysis scheme, where the initial velocity estimate uses only relatively low frequencies (requiring fewer samples) and apply it sequentially in depth starting with the upper layers first. We note that, in contrast, RTM requires the entire domain to be modelled in each migration step.

RESULT

To test our method we used the velocity and density model in Figure 1 a and b. We generated synthetic data by forward modeling 80 shots equally spaced along the surface 200 meters apart, using a Ricker wavelet with dominating frequency 6 Hz. We recorded the forward modeled data at 641 equally spaced receivers along the surface.

The synthetically generated data was migrated using the method in this paper with the migration velocity in Figure 1c. The oridiginal migrated image is shown in Figure 2a. We note the low-frequency artifacts above the salt body (also characteristic for RTM migrated data). In Figure 2b we show the migrated image after applying a Laplacian filter to the data to dampen the low-frequency artifacts. In Figure 2c we show a denoised migrated image. We note that the steep reflectors and overturned targets are well imaged with our approach.

Finally, we show one of the angle gathers along the line $x = 8000$ in Figure 3.

CONCLUSION

The migration algorithm of this paper allows us to use the incident and scattered wavefields extracted from the measured data to compute depth extrapolated wave fields and, as a result, form an image to recover steep targets. We note the flexibility of the algorithm allowing us to combine it with velocity analysis. We plan to develop such migration velocity analysis and associated inversion algorithms in the near future.

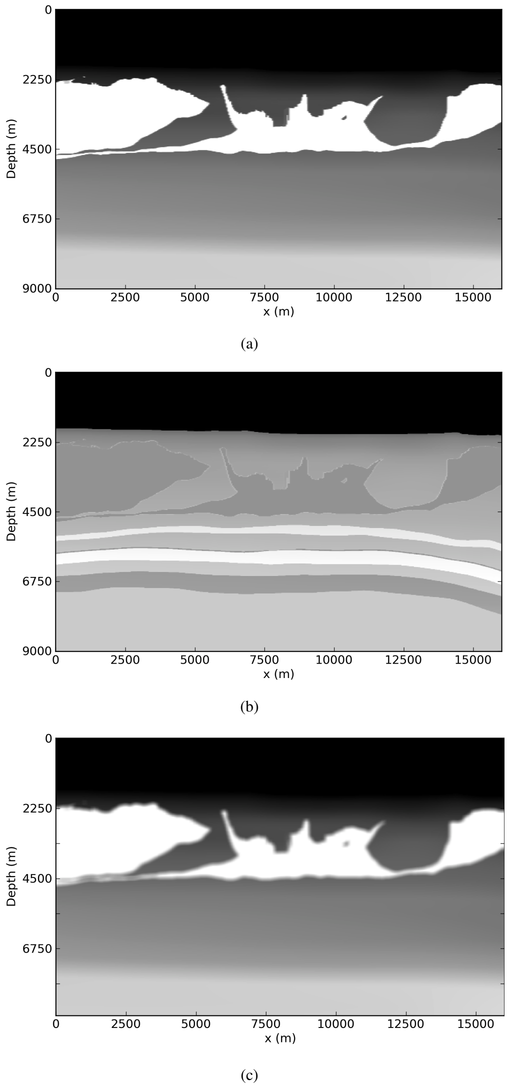

Figure 1: (a) Original velocity model. (b) Original density model. (c) Velocity model used for migration.

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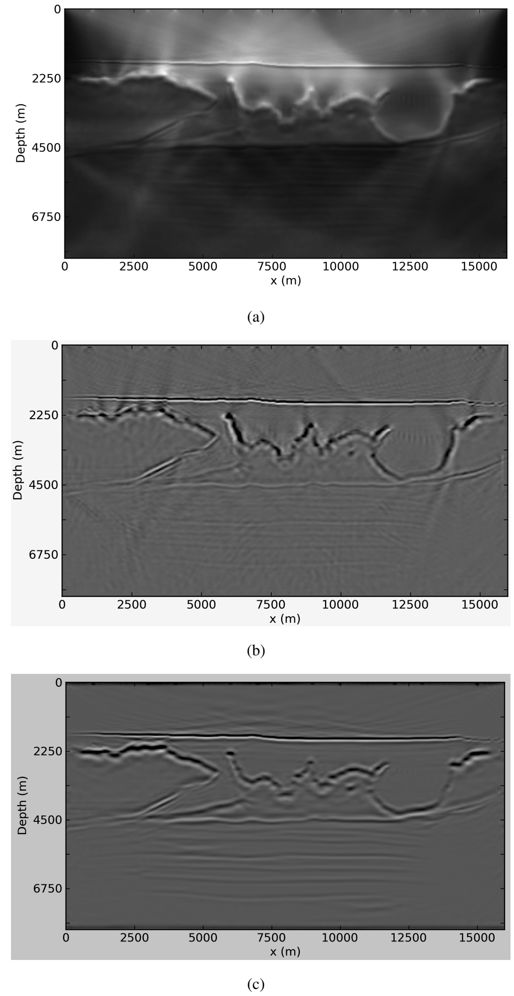

Figure 2: (a) Migrated image. (b) Migrated image after applying a Laplacian filter. (c) Denoised migrated image.

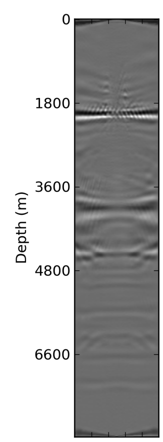

Figure 3: Angle gather along $x = 8000$ m.