Layer Stripping Migration Velocity Analysis Using Accurate Survey Sinking

SUMMARY
We present a survey sinking based framework for residual migration velocity analysis. By using a migration algorithm in the temporal frequency domain, our migration algorithm is efficient and allows us to pick velocity perturbations directly. Picking velocity perturbation directly removes the issues associated with estimating velocity perturbations from indirect picking parameters, such as time lags. Our methodology is based on a wave-equation migration method that combines high accuracy with computational efficiency. By relying on survey sinking migration, we avoid the problem of estimating the source wavelet, which is a rarely mentioned problem associated with RTM and shot-gather migration. Our survey sinking framework allows us to robustly estimate the velocity sequentially in relatively thin layers.

INTRODUCTION
In order to produce a useful image of the subsurface, migration algorithms require an accurate estimate of the wave propagation velocity. Inaccurate velocity estimates typically result in a degraded image, and the effect of poor velocity estimates rapidly degrades the image at increasing depths.

To address this, a wide variety of Migration Velocity Analysis (MVA) tools have been proposed. The strategy is to use some type of localized quality (“focusing”) measure applied to the migrated image and, based on such measure, estimate how the migration velocity should be modified to generate a more geophysically probable image.

In MVA it is natural to use some type of extended imaging condition (see, e.g., Sava and Vlad, 2008, and Sava and Vasconcelos, 2010). A common way of extending the imaging condition is to study the migrated image for a range of time shift lags (temporal offsets), and pick the time lag that locally generates the best-focused migrated image. The use of time lags is popular as it typically can be applied efficiently without having to re-migrate the data. Migrating the data can be computationally expensive for accurate wave-equation based migration algorithms. For this reason using time lags is often the method of choice for MVA in connection with wave-equation based migration methods.

The major disadvantage of using time lags (or spatial offsets) as parameter for MVA, is that the connection between the time lag and the velocity perturbation can be complicated to estimate in a complex medium (Yang and Sava, 2001). Even if a relation between time lag and velocity perturbation can be formally established, the resulting equations can be ill-conditioned, and lead to severe numerical problems. Furthermore, such solutions are typically only valid if the time lag (and the associated velocity perturbation) is small.

To circumvent the computational issues associated with computing velocity perturbation from time lags, it is desirable to apply MVA directly to the velocity perturbation, rather than to an indirect parameter such as time lag. The apparent disadvantage of such a direct approach is that one has to migrate the data once for each velocity perturbation. When using wave-equation based migration algorithms this may result in significant computational complexity. However, if the computational complexity can be reduced, MVA applied directly to velocity perturbation carries significant advantages, and dramatically simplifies its implementation.

Since velocity errors accumulate over depth, it is important to establish a good velocity model in upper layers before migrating data to deeper depths. To this end, the technique of layer stripping is often used, where MVA is applied to upper layers first (Wang et al., 2006, and Lau, et al., 2013).

In this paper, we develop a methodology that satisfies the following criteria:

1. Use an accurate migration algorithm that works well even in complex media.
2. Use a migration algorithm that can be efficiently combined with MVA within a layer-stripping approach.
3. Use a migration algorithm that is sufficiently fast to allow MVA to be applied directly to the velocity perturbation. In particular, the algorithm should be easily parallelizable.

In order to satisfy the first criterion we have to use a wave-equation based migration algorithms. Unfortunately, RTM is computationally expensive, and cannot be easily used in a layer stripping framework and, thus, does not satisfy criteria 2 and 3. On the other hand, a naively implemented frequency based approach does not handle complex media well, and cannot propagate (near) horizontal waves with sufficiently high accuracy. We therefore select an approach based on the spectral-projector framework in Sandberg and Beylkin (2009), which provides high accuracy in complex media, for all propagation angles, and naturally fits into a layer-stripping approach MVA.

Additionally, a frequency based method has the advantage of a multi-scale approach, where we first migrate the data using only the lowest temporal frequencies. Lower temporal frequency data can be discretized on a coarse spatial grid, therefore leading to significant computational savings and a small memory footprint. These computational savings allow us to satisfy criterion 3 above, that is, to apply MVA directly to the velocity perturbation.

With regard to using shot-gather migration or survey sinking migration, we note that both choices are possible. Shot-gather
Layer Stripping Migration Velocity Analysis Using Accurate Survey Sinking

migration has the advantage that in certain cases even overturned events can be imaged with accuracy similar to that of RTM (Sandberg, et al., 2010). Survey sinking on the other hand, has the advantage of not relying on cross-correlation, thus avoiding a convolution with a typically unknown wave form, a consideration typically overlooked in the literature where it is often (and wrongly) assumed that the two types of migration are equivalent. Even if such a wave form can be estimated, the actual wave form used during data acquisition, often varies over both time and space. RTM also suffers from the same disadvantage as shot-gather migration, as the resulting image depends on the waveform selected for the source wave field. On the other hand, survey sinking does not require the generation of any source wave field and is well-suited for layer stripping, as it propagates the survey in depth, rather than time.

We use the survey sinking method combined with the generation of angle gathers to perform residual migration velocity analysis directly with respect to the velocity perturbation. We first outline the workflow followed by a description of associated algorithms. We then demonstrate our approach on a synthetic example and illustrate our methodology on a real seismic data set.

METHODOLOGY

Migration Algorithm
We use a migration algorithm suitable for layer stripping that combines high accuracy with computational efficiency.

We use the migration algorithm outlined in Sandberg and Beylkin (2009). This wave equation based algorithm uses spectral projectors to suppress only the evanescent modes without affecting the propagating modes. Thus, it overcomes the problem of loss of accuracy for near-horizontally propagating waves typical of common implementations of wave-equation migration algorithms in the temporal frequency domain. Furthermore, this algorithm remains accurate in a complex medium.

During the migration, we record the image for a range of spatial offsets between sources and receivers, effectively generating so called Offset Domain Common Image Gathers (or OD-CIG, hereafter referred as offset gathers). See Biondi (2006) for further details.

Angle Gather Computation
We can generate Angle Domain Common Image Gathers (or ADCIG, hereafter referred to as angle gathers) from the offset gathers recorded during the survey sinking migration. These gathers can be computed either using a Fourier-based approach, or by computing a slant stack (for details, see Sava and Fomel, 2003). The computational time for generating the angle gathers is negligible compared to the cost of migration.

It is only necessary to generate angle gathers at selected horizontal locations, e.g., every 100 meters.

Workflow Overview
In order to avoid accumulation of velocity errors, it is critical to perform MVA iteratively in (relatively) thin layers of the domain (layer stripping). Once we have obtained satisfactory focusing of events in the current layer, we use the survey sunk to the bottom of the current layer as a starting survey for the next layer. Although layers at increasing depth must be processed serially, we can parallelize migration computations within each layer. Parallelization can be easily implemented with respect to either velocity perturbation, temporal frequency, or both. The speed advantage that parallelization offers enables us to use a highly accurate wave-equation based migration algorithm. We can also achieve significant speed savings by only migrating the lowest temporal frequencies necessary to form reliable angle gathers. The frequency range needed for this purpose is typically considerably lower than the full frequency content we use to generate the final migrated image.

Let us assume that we have a starting migration velocity model, referred to as $v_{orig}$. In this paper, (for simplicity of exposition) we perturb the migration velocity by rescaling it within the current layer by a scalar factor $\alpha$. However, we note that other types of velocity parameterizations can be used as well. For example we can use a Fourier- or wavelet-decomposition of the velocity model, and then perturb the modes of such decomposition. We note that such approach may be appropriate in media with large variations.

Within each layer, the procedure can be summarized as follows:

1. For each velocity perturbation $\alpha_k$, $k = 1, 2, ..., n$:
   (a) Compute the perturbed velocity model
      
      $$v_k = \alpha_k v_{orig}.$$ 
   (b) Migrate the survey using velocity $v_k$, and using only the lowest temporal frequency modes necessary to form reliable angle gathers. Record the offset gathers at each depth step.
   (c) Compute angle gathers from the offset gathers generated during migration.

2. Generate angle gather montages, where each montage contains the angle gathers for all velocity perturbations at selected horizontal locations.

3. At each selected horizontal location, record the position and velocity perturbation that generate flat events in the angle gathers.

4. Based on the horizontal location and corresponding picks in depth and velocity, use interpolation to update the starting velocity migration model $v_{orig}$.

5. Repeat Steps 1-5 as needed, by perturbing the updated migration velocity model $v_{orig}$.

Once a layer has been iteratively improved to satisfaction, we use the survey recorded at the bottom of the current layer as the starting survey for the subsequent layer.

We note that one important advantage of using survey sinking instead of RTM, is that it allows us to only migrate the (lower) temporal frequencies needed to create useful angle gathers.
EXAMPLES

Synthetic Example
We first illustrate the methodology on a synthetic example. The original model is shown in Figure 1a. We generated synthetic source gathers using a highly accurate pseudo-spectral modeling algorithm. In order to illustrate our methodology we migrate the data using the incorrect migration velocity model in Figure 1b. The migration velocity model is piece-wise constant, while our true model contains smooth variations throughout the domain.

![Figure 1: a) Original model. b) Migration velocity model.](image)

Even though in practice we would perform the velocity analysis in thin layers, we find it informative to illustrate the procedure on the full model, as we can then see how the angle gathers become less useful for deeper depths. In Figure 2a we show the migrated image using the incorrect velocity model in Figure 1b.

![Figure 2: a) Migrated image using the velocity in Figure 1b. b) Montage of angle gathers recorded for a range of velocity perturbations along a single location at x = 300 (marked by the vertical line in Figure 2a).](image)

We next migrate the data using a number of scaled versions of the velocity model $v_{\text{orig}}$ in Figure 1b. In Figure 2b we show a montage of angle gathers recorded at $x = 300$ using only temporal frequencies up to 24Hz, compared to the full range of up to 90 Hz for the recorded survey. We display one angle gather for each velocity perturbation $\alpha = 0.9v_{\text{orig}}, 0.92v_{\text{orig}}, ..., 1.1v_{\text{orig}}$. For an event imaged with a locally correct migration velocity, we expect to see a flat event in the angle gather.

We make a number of observations from the angle gather mon-
Layer Stripping Migration Velocity Analysis Using Accurate Survey Sinking

tage in Figure 2b. We first notice that the false reflector caused by the multiple reflection between the surface and first interface does not result in a flat event in the angle gather for any velocity. This is typically the case for multiple reflections: no (reasonable) velocity will give a flat event in the angle gathers. A similar phenomenon can be observed for events deeper down in the image. We note that it becomes increasingly difficult to find flat events in the angle gathers for the reflections deeper down in the migrated image. This is due to the fact that the velocity is not only incorrect locally, but the event also suffers from the accumulated velocity inaccuracies along the wave path.

Seismic Data Example

We next illustrate our methodology on a 2D land survey acquired in South Texas using a Vibroseis source. The zone of interest was the EagleFord interval above the Buda carbonate. An initial velocity model $v_{\text{orig}}$ was generated using Kirchoff-based tomography, and is shown in Figure 3a. In Figure 3b we show the migrated image using temporal frequencies up to 100 Hz.

![Velocity model](image1)

Figure 3: a) Velocity model. Range: 1800 m/s (blue) to 6000 m/s (red) b) Migrated image. The domain covers an area of 6000 meters in depth and 12000 meters horizontally.

We now illustrate our velocity analysis approach by generating angle gathers for the upper 750 meters of the region. For the purpose of velocity analysis, we used frequencies up to 50 Hz.

We migrate the data for this portion using velocity perturbations $\alpha = 0.9v_{\text{orig}}, 0.92v_{\text{orig}}, ..., 1.1v_{\text{orig}}$. In Figure 4a we show the migrated section (covering the upper 750 meters of the full image domain). Figure 4b displays a single composite angle gather across these velocity perturbations at the location marked in Figure 4a. The gather is sorted left to right by increasing perturbation and absolute angle. Initially, events appear at different depths increasing with the perturbation. Event picks are made interactively in top-down order, where an event appears the most flat. A depth squeeze/stretch is performed so that events up to the deepest pick are aligned.

![Migrated image](image2)

Figure 4: (a) Migrated image for the upper 750 m depth section. (b) Angle gather display from interactive picker with interpolated residual velocity correction along the black line.

We note that although the initial velocity model is fairly accurate, we still observe deviations even in this uppermost layer. The effects of such velocity inaccuracies will have an increasingly deteriorating effect on reflectors further down in the image.

CONCLUSION

We have described a direct and stable method for residual migration velocity analysis. The methodology is based on a wave-equation migration method that combines high accuracy with computational efficiency, even in complex media. By using survey sinking migration, we avoid problems of estimating the source wavelet, which is a rarely mentioned problem associated with RTM and shot-gather migration. Our migration algorithm is sufficiently fast to allow velocity analysis with respect to velocity perturbations, which removes problems associated with velocity analysis based on extended imaging conditions using time lags or spatial offsets.

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